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XVI.

ATMOSPHERIC REFRACTION.

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PART II.

THE observations described in Part I. relate exclusively to the refraction of the portion of the air between two objects on the surface of the earth. In astronomical observations we have to consider the effect of the entire column of air traversed by the light from an object outside the earth's atmosphere until it reaches the observer. The variation of this quantity, and the effect of local causes upon it, is an important source of error in many astronomical observations. For instance, the systematic differences in the declinations of the southern stars, as determined at different observatories, may be due to different refractions near the northern and southern horizons. The study of this matter has usually been left to the large alt-azimuths and transit-circles to be found in an astronomical observatory. From the fixed position of these instruments it is not easy to vary the conditions as much as might be desired. We are therefore ignorant of the variations of the refraction in different azimuths, or the effect upon it of the proximity of large masses of water, of forests, or of snow-covered mountains. Even its variations in different parts of the world are but little known, and it is usual to employ the refraction tables of Bessel, or those of the Pulkova Observatory, under the most varied conditions of climate or local surroundings. The micrometer level seems to be especially adapted to measuring the atmospheric refraction, and it is hoped that the observations described below will show that it is quite practicable for a traveller to determine this quantity at any point where the results are likely to be of interest. Not the least interesting of the results which may be thus obtained is the determination of the law regulating the refraction at great elevations.

The use of the micrometer level is limited to altitudes of three or four degrees; but within these limits the refraction and its uncertain variations are so large that angular measurements of great accuracy are not required. The only instruments needed are the micrometer level, a chronometer with some means of determining its error and rate, a barometer, and a thermometer.

The observations described below consisted in a series of determinations of the corresponding altitudes and times at which the sun or a star gradually approached the horizon. A complete observation consisted in observing the temperature and pressure of the air, and determining the error of the chronometer by comparing it with a standard clock whose error was known. The micrometer level was then placed in position on the west balcony of the dome of the Harvard College Observatory, and its collimation and level constants determined as described on page 269. The telescope was next turned nearly in the direction in which the sun or star would set, and several readings of the level taken in various azimuths. A series of measures was then made of the apparent altitude of the object as it approached the horizon, and the corresponding times. Finally, the preliminary measures, or such a portion of them as seemed to be essential, were repeated. When the sun was observed, the settings were first made on its lower limb until it disappeared below the horizon, and then the upper limb was measured until it also disappeared. For night observations, a fine needle was inserted in the field, and this formed a dark bar, which was visible against the sky without the necessity of a special field illumination.

A summary of the measures is contained in Table VIII., which gives, in successive columns, a number for reference, the date, the approximate Greenwich mean time, and the object observed. These are followed by the number of settings made, the corrected atmospheric pressure in inches, and the temperature in Fahrenheit degrees.

Series 2 to 8 inclusive were made by Mr. D. B. Pratt, the others by myself. Of the settings, 329 were made on *a Bootis*, 294 on the upper limb of the sun, and 122 on the lower limb, or 745 in all.

The value of the instrumental constants employed are given in Table IX. The successive columns give a number for reference, the date, and the Greenwich mean time. The next four columns give the apparent elevation of the object observed with the instrument placed in its four different positions, O_p, O_v, E_p, and E_v. The mean of these four readings gives the apparent height of the object. The excess of either of the four readings over the mean gives the correction to

TABLE VIII. — SUMMARY OF SERIES.

No.	Date.	G. M. T.	Object.	No. Obs.	Barometer.	Ex. Therm.
	1885.	h.			in.	°
1	Aug. 8	12.7	Sun	99	30.072	69.2
2	" 11	11.8	"	38	29.955	76.1
3	" 14	11.7	"	34	29.550	78.2
4	" 15	11.7	"	26	29.867	66.6
5	" 22	11.6	"	60	29.836	76.6
6	Sept. 2	11.2	"	76	30.034	55.0
7	" 2	15.3	α Bootis	39	30.034	55.0
8	" 3	11.2	Sun	83	30.012	59.9
9	" 20	14.2	α Bootis	51	30.222	48.0
10	" 21	14.1	"	45	29.991	52.1
11	Oct. 10	12.8	"	50	30.075	52.4
12	" 17	12.3	"	55	29.988	50.8
13	" 25	11.8	"	89	30.149	44.2

be applied to measures made in the corresponding position of the instrument. The position of the instrument actually employed was O_p, except in the observations to which Nos. 20 to 31 relate, when it was O_v. The corresponding excess is given in the next column. The last two columns give the values of the collimation and level error, in seconds of arc.

During September the level was used in some geodetic observations among the mountains of New Hampshire and Vermont. Nos. 20 to 25 were taken from the top of Mt. Moosilauk, and Nos. 26 to 31 from Mt. Mansfield. Although the instrument was carried in a wagon over rough mountain roads, the effect on its constants seemed to be inappreciable. The level error, as shown in the last column, did not appear to change perceptibly during the entire series. The collimation at first underwent a singular change which may have been due to a looseness of the screws holding the reticule. No change appears to have taken place during a single series of observations, since the collimation was always substantially the same before and after it, that is, in the pairs of measures made upon the same date. The effect is therefore eliminated in the final results. No change appears to have taken place after September 3.

Nos. 11, 15, 16, 19, 32, 33, 34, 35, 36, and 37 relate to the needle used to observe *α Bootis;* in the other cases the intersection of the cross-wires was observed. The interval between them was about 85 divisions, which affects the collimation, but not the level, to this extent.

In No. 10 the reading of E_v has been assumed to be in error by one turn of the screw; otherwise, the observed value would be —46.7.

TABLE IX. — CONSTANTS OF INSTRUMENT.

No.	Date.	G.M.T.	*Op.*	*Ov.*	*Ep.*	*Ev.*	Ex.	Col.	Level.
		h.						″	″
1	Aug. 8	11.6	— 16.1	— 21.7	+ 22.5	+ 15.1	+ 2.7	+ 45	0
2	" 8	12.7	— 17.1	— 18.8	+ 20.0	+ 15.7	+ 0.8	+ 21	0
3	" 11	11.5	— 18.0	— 19.0	+ 19.3	+ 17.0	+ 0.3	+ 11	— 3
4	" 11	11.8	— 18.0	— 19.6	+ 19.7	+ 18.0	+ 0.8	+ 11	0
5	" 14	11.4	— 21.0	— 13.4	+ 14.2	+ 22.1	— 3.3	— 54	+ 7
6	" 14	11.7	— 21.4	— 14.0	+ 13.0	+ 21.5	— 3.9	— 56	— 3
7	" 15	11.7	— 28.4	— 12.5	+ 8.6	+ 25.0	— 9.8	— 116	—25
8	" 22	11.1	— 42.9	— 19.0	+ 19.7	+ 42.6	—11.9	— 168	+ 1
9	" 22	11.6	— 41.9	— 19.2	+ 19.8	+ 42.8	—11.0	— 160	+ 6
10	" 29	11.2	— 47.5	— 92.2	+ 93.1	+ 53.3	+24.0	+ 294	+24
11	" 31	11.4	— 21.5	—161.2	+160.6	+ 23.0	+70.1	+ 967	+ 3
12	Sept. 1	11.0	— 35.5	— 10.0	+ 9.7	+ 33.4	—13.3	— 172	— 9
13	" 2	10.8	— 36.0	— 10.7	+ 9.6	+ 33.8	—13.5	— 173	—11
14	" 2	11.2	— 36.7	— 7.2	+ 7.8	+ 36.4	—14.7	— 193	+ 1
15	" 2	14.7	+123.0	— 17.4	+ 15.7	—123.5	+69.6	+ 976	— 9
16	" 2	15.3	+123.0	— 17.2	+ 30.6	—131.5	+71.3	+1054	+14
17	" 3	10.7	— 28.2	— 17.4	+ 16.5	+ 26.5	— 6.0	— 73	— 9
18	" 3	11.2	— 26.2	— 16.7	+ 17.2	+ 28.4	— 4.1	— 73	+10
19	" 3	16.1	+130.1	— 26.3	+ 27.0	—122.0	+80.4	+1066	+31
20	" 6	3.6	—148.3	—149.3	+153.9	+150.1	+ 2.1	+ 17	+22
21	" 6	6.0	—150.1	—153.0	+154.5	+150.6	+ 1.9	+ 24	+ 7
22	" 7	1.6	+ 75.1	+ 70.7	— 70.6	— 74.8	+ 2.3	— 31	+ 1
23	" 7	4.8	—176.5	—180.5	+181.4	+177.9	+ 2.6	— 27	+ 9
24	" 7	7.9	—177.4	—180.5	+180.6	+177.5	+ 1.6	+ 22	0
25	" 7	11.8	—112.8	—116.2	+116.9	+112.0	+ 1.4	+ 29	0
26	" 10	4.0	—215.9	—211.4	+212.4	+216.3	— 1.9	+ 29	+ 6
27	" 10	6.6	—177.9	—181.6	+181.6	+178.3	+ 1.9	— 25	+ 1
28	" 12	8.0	+ 79.0	+ 74.3	— 73.2	— 79.8	+ 2.4	+ 39	+ 1
29	" 12	10.8	+ 79.2	+ 74.0	— 75.1	— 78.5	+ 2.5	— 31	— 1
30	" 15	1.9	+ 77.3	+ 74.3	— 73.7	— 78.0	+ 1.5	+ 25	0
31	" 16	8.4	+ 76.6	+ 74.6	— 73.4	— 76.6	+ 1.3	+ 18	+ 4
32	" 20	14.2	—108.0	—282.3	+283.8	+112.3	+88.6	+1206	+20
33	" 21	14.1	+ 54.3	—117.6	+121.3	— 51.1	+87.7	+1201	+24
34	" 27	12.4	—112.0	—283.2	+283.9	+113.6	+85.9	+1190	+ 7
35	Oct. 10	12.8	+140.5	— 29.4	+ 33.2	—138.4	+86.4	+1192	+22
36	" 14	11.4	+176.6	+ 5.2	— 6.0	—174.4	+83.6	+1186	+ 6
37	" 17	12.3	+125.6	— 45.3	+ 47.4	—123.7	+86.4	+1193	+14
38	" 25	11.8	—112.0	—284.5	+282.5	+115.1	+86.5	+1186	+ 4

These results have next been compared with Bessel's refractions by the aid of Table X. This gives the mean refraction for altitudes of every 100″ from the horizon to 5°, for a temperature of 48°.8 and a barometric pressure of 29.6 inches. The altitude corresponding to any refraction given in the table is found by adding the argument at the top of the column to that given in the first column, all the quantities being expressed in seconds of arc. Thus the refraction 1815″ corresponds to the altitude 1400″, 618″ to 16900″, etc.

After applying the corrections for temperature and pressure of the air to each observation, the residuals have been found by subtracting

TABLE X. — Mean Refractions.

Alt.	00″	100″	200″	300″	400″	500″	600″	700″	800″	900″
00″	2094	2073	2052	2031	2010	1990	1969	1949	1929	1909
1000	1890	1871	1852	1834	1815	1797	1779	1762	1744	1727
2000	1710	1693	1677	1660	1643	1627	1612	1597	1581	1566
3000	1550	1535	1521	1507	1493	1479	1465	1452	1439	1426
4000	1413	1400	1387	1375	1363	1351	1339	1327	1316	1305
5000	1295	1284	1273	1262	1251	1241	1231	1221	1211	1202
6000	1192	1183	1174	1165	1156	1147	1138	1130	1122	1114
7000	1106	1097	1089	1081	1073	1065	1058	1051	1043	1036
8000	1029	1022	1015	1008	1001	994	987	980	974	968
9000	961	955	948	942	935	929	923	917	911	905
10000	900	894	888	882	877	871	866	861	855	850
11000	845	840	834	829	824	819	814	810	805	800
12000	795	791	786	782	777	773	768	764	760	756
13000	752	748	744	740	736	733	729	725	721	717
14000	714	711	707	703	699	695	692	688	685	682
15000	678	675	672	669	665	662	659	656	653	650
16000	646	643	640	637	634	631	627	624	621	618
17000	615	612	609	606	603	600	597	594	592	589

TABLE XI. — Number of Observations.

Alt. ° ′	1.	2.	3.	4.	5.	6.	8.	9.	10.	11.	12.	13.
0 20	..	..	..	..	..	1	..	..	..	..	..	..
30	..	..	..	..	..	1	5	..	..	..	..	..
40	8	..	..	..	..	4	8	2	2	..	..	..
50	18	6	7	4	..	5	8	3	2	..	..	..
1 0	16	6	6	4	1	7	6	2	3	..	..	..
10	8	1	6	4	8	4	4	3	2	..	..	2
20	8	..	..	3	5	4	4	2	2	..	..	4
30	8	..	..	4	2	3	4	3	3	..	..	5
40	8	3	..	5	3	3	4	2	2	4	..	5
50	5	6	3	2	3	3	4	2	2	4	..	6
2 0	..	4	4	..	3	3	4	3	3	5	2	5
10	..	4	3	..	4	3	4	2	1	5	2	6
20	..	2	1	..	3	3	4	2	3	5	5	6
30	..	3	..	..	3	3	4	2	2	3	4	6
40	..	2	..	..	4	3	4	1	3	1	4	6
50	8	1	..	..	3	3	4	3	2	..	3	6
3 0	8	..	..	..	2	3	4	2	2	1	4	6
10	4	..	..	..	3	4	4	2	2	5	4	5
20	..	..	1	..	3	3	3	2	2	4	4	6
30	..	..	2	..	3	3	1	1	2	4	5	5
40	..	..	1	..	3	4	..	2	2	4	3	5
50	..	..	..	..	2	3	..	2	2	2	2	3
4 0	..	..	..	..	2	3	..	2	1	2	3	2
10	..	..	..	..	..	..	..	2	..	1	2	..
20	..	..	..	..	..	..	..	1	..	..	2	..
30	..	..	..	..	..	..	..	2	..	..	1	..
40	..	..	..	..	..	..	..	1	..	..	3	..
50	..	..	..	..	..	..	..	..	..	..	1	..

from them the refraction as given by Bessel. These residuals are arranged in groups in Tables XI. and XII. Each group extends over 10′, its central point being given in the first columns. Table XI. gives the number of residuals contained in each group, and Table XII. their mean value. The corresponding number of the series is given at the top of each column. The measures of the upper and lower limbs of the sun are combined, as there seems to be no systematic difference between them. Series 7 is omitted, since there is an error in the number of turns of the micrometer screw, or in the number of minutes in the observed times.

The results of these two tables are combined in Table XIII. The successive columns give the altitude, the corresponding refraction according to Bessel, the total number of observations of the sun, and of *α Bootis.* The next two columns are derived from Table XII., and give the means of the residuals contained in that table relating to the sun, and the means of those relating to *α Bootis.*

TABLE XII. — MEAN RESIDUALS.

Alt.	1.	2.	3.	4.	5.	6.	8.	9.	10.	11.	12.	13.
° ′												
0 20	..	..	..	..	..	—173	..	..	..	..	..	..
30	..	..	..	..	..	—131	— 92	..	..	..	..	..
40	—121	..	..	..	..	—129	—117	+40	—29	..	..	..
50	—136	— 96	—65	—75	..	—137	—119	+16	—24	..	..	..
1 0	—142	—119	—93	—78	—75	—134	—110	—26	—26	..	..	..
10	—139	—146	—78	—52	—89	—119	—104	—23	—21	..	..	—28
20	—114	..	..	—56	—85	—104	— 98	+ 1	—58	..	..	—46
30	—123	..	..	—64	—74	— 51	— 80	0	—29	..	..	—26
40	—128	— 40	..	—79	—64	—107	— 79	+ 8	—58	—16	..	—29
50	—146	— 72	—74	—91	—59	—117	— 88	+22	—28	—13	..	—35
2 0	..	— 41	—86	..	—59	—110	— 92	+15	—25	0	—44	—33
10	..	— 63	—64	..	– 54	— 66	— 82	+20	—14	— 6	—58	—27
20	..	— 38	—53	..	—43	— 25	— 78	+15	—20	—16	—45	—29
30	..	— 29	..	..	—36	—111	— 61	+37	+ 4	— 5	—40	—28
40	..	— 66	..	..	—49	— 85	— 59	+26	— 9	+ 2	—39	—21
50	— 88	— 39	..	..	—20	— 81	— 60	+20	— 4	..	—32	— 6
3 0	— 90	..	..	..	—37	— 78	— 61	+29	— 2	— 9	—32	+ 2
10	— 94	..	..	..	—29	— 82	— 70	+65	— 2	+ 6	—34	+ 5
20	..	..	—59	..	—20	— 66	— 60	+26	+12	+11	—20	+10
30	..	..	—32	..	—20	— 51	— 61	+50	+ 9	— 6	—21	+12
40	..	..	—62	..	—14	— 60	..	+66	+26	+14	— 3	+24
50	..	..	..	..	—10	— 68	..	+34	+12	+ 8	+11	+18
4 0	..	..	..	..	—12	— 67	..	+52	— 2	+16	+ 9	+ 8
10	..	..	..	..	..	..	..	+62	..	+19	—48	..
20	..	..	..	..	..	..	..	+59	..	..	+14	..
30	..	..	..	..	..	..	..	+60	..	..	+14	..
40	..	..	..	..	..	..	..	+56	..	..	+33	..
50	..	..	..	..	..	..	..	..	..	..	+22	..

TABLE XIII.

Alt.	Bessel Ref.	No.		Mean.	
		S.	a.	S.	a.
° ′					
0 20	1852	1	..	—173	..
30	1744	6	..	—111	..
40	1643	20	4	—122	+ 6
50	1550	48	5	—105	— 4
1 0	1465	46	5	—107	—26
10	1387	35	7	—104	—24
20	1316	24	8	— 91	—34
30	1251	21	11	— 78	—18
40	1192	26	13	— 83	—24
50	1138	26	14	— 92	—14
2 0	1089	18	18	— 78	—17
10	1043	18	16	— 66	—17
20	1001	13	21	— 47	—19
30	961	13	17	— 59	— 6
40	923	13	15	— 65	— 8
50	888	19	14	— 58	— 6
3 0	855	17	15	— 66	— 2
10	824	15	18	— 69	+ 8
20	795	10	18	— 51	+ 8
30	768	9	17	— 41	+ 9
40	744	8	16	— 45	+25
50	721	5	11	— 39	+17
4 0	699	5	10	— 40	+17
10	678	..	5	..	+11
20	659	..	3	..	+36
30	640	..	3	..	+37
40	621	..	4	..	+44
50	603	..	1	..	+22

The fact noticed by Argelander, that the refraction derived from the setting sun is less than that of a star is well shown in this table. The difference amounts to one or two minutes of arc.

In this investigation the value of one division of the screw in seconds must be known with accuracy. It was therefore redetermined August 6, 1885, with the same result, 13″.95, as that originally found.

This paper is intended to show that the micrometer level is capable of giving useful results where a larger instrument has generally been considered necessary. Its portability, and the rapidity with which observations may be made by it, adapt it especially to the wants of travellers, and would permit the accumulation of valuable information regarding the atmospheric conditions of comparatively inaccessible points. If required, much greater accuracy could doubtless be attained than is indicated by the stellar observations described above. The instrument was mounted on a wooden balcony, and the times were only taken to whole seconds. Instead of moving the telescope each

time, it might be better to have a series of lines in the field, and observe the transits over each, as in a meridian instrument. The advantages of the two forms of instrument employed, attaching the level to the telescope or to the wyes, will vary with the surrounding conditions. The principal objection to the second method is the time required to determine the constants. This may be done almost equally well when the level is attached directly to the telescope, by taking reciprocal readings from two points one or two hundred yards apart. The variations of the instrumental constants will also doubtless be less with this form of instrument. In either case, if many observations are to be taken from a given station, it is advisable to determine once for all the absolute altitude of some convenient object, and refer everything to that, like the meridian mark of a transit instrument.